Biot Savart Law

Objectives

- After going through this lesson, the learners will be able to:
- Interpreting the concept of the magnetic field produced by a moving charge
- Know the direction of the magnetic field produced by a moving charge (Electric Current)
- Identify factors that would determine the strength of the magnetic field produced at a point due to the current carrying conductor
- Derive an expression of Biot Savart Law
- Differentiate between Coulombs' law with Biot Savart law

Content Outline

- Unit syllabus
- Module wise distribution
- Words you must know
- Introduction
- Concept of Magnetic field
- Oersted's Experiments
- Ampere's Swimming Rule
- Maxwell's Cork Screw Rule
- Right hand thumb Rule
- Biot-Savart Law
- Comparison of Biot-Savart Law with Coulomb's Law
- Examples based on Biot-Savart Law
- Summary

Unit Syllabus

Chapter-4: Moving Charges and Magnetism

Concept of magnetic field, Oersted's experiment.

Biot-Savart law and its application to the current carrying circular loop.

Ampere's law and its applications to infinitely long straight wire. Straight and toroidal solenoids, Force on a moving charge in uniform magnetic and electric fields. Cyclotron.

Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field; moving coil galvanometer-its current sensitivity and conversion to ammeter and voltmeter.

Chapter-5: Magnetism and Matter

Current loop as a magnetic dipole and its magnetic dipole moment. Magnetic dipole moment of a revolving electron. Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. Torque on a magnetic dipole (bar magnet) in a uniform magnetic field; bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements.

Para, dia and ferro - magnetic substances, with examples. Electromagnets and factors affecting their strengths. Permanent magnets.

Module Wise Distribution-10 Modules

The above unit is divided into 10 modules for better understanding.

Module 1	● Introducing moving charges and magnetism ● Direction of magnetic field produced by a moving charge ● Concept of Magnetic field ● Oersted's Experiment ● Strength of the magnetic field at a point due to current carrying conductor ● Biot-Savart Law ● Comparison of coulomb's law and Biot Savart's law
Module 2	● Applications of Biot- Savart Law to current carrying circular loop, straight wire ● Magnetic field due to a straight conductor of finite size ● Examples
Module 3	● Ampere's Law and its proof ● Application of ampere's circuital law: straight wire, straight and toroidal solenoids.

	● Force on a moving charge on a magnetic field ● Unit of magnetic field ● Examples
Module 4	● Force on moving charges in uniform magnetic field and uniform electric field. ● Lorentz force ● Cyclotron
Module 5	● Force on a current carrying conductor in uniform magnetic field ● Force between two parallel current carrying conductors ● Definition of ampere
Module 6	● Torque experienced by a current rectangular loop in uniform magnetic field ● Direction of torque acting on current carrying rectangular loop in uniform magnetic field ● Orientation of a rectangular current carrying loop in a uniform magnetic field for maximum and minimum potential energy
Module 7	● Moving coil Galvanometer- ● Need for radial pole pieces to create a uniform magnetic field ● Establish a relation between deflection in the galvanometer and the current ● Current sensitivity ● Voltage sensitivity ● Conversion to ammeter and voltmeter ● Examples
Module 8	● Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. ● Torque on a magnetic dipole in a uniform magnetic field. ● Explanation of magnetic property of materials
Module 9	● Dia, Para and ferromagnetic substances

	● Electromagnets and factors affecting their strengths, permanent magnets.
Module 10	● Earth’s magnetic field and magnetic elements.

Module 1

Words You Must Know

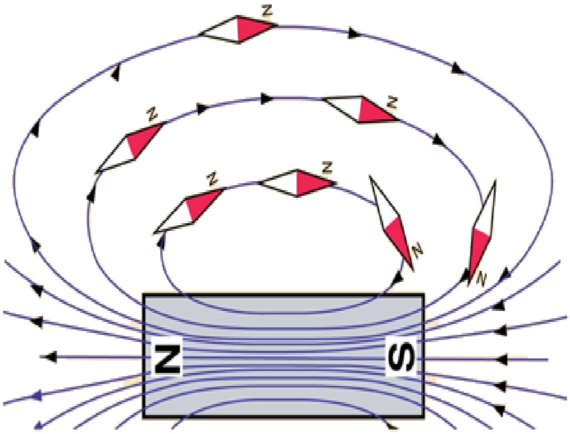
- **Coulomb’s law:** The force of attraction or repulsion between two point charges is directly proportional to the product of two charges (q_1 and q_2) and inversely proportional to the square of the distance between them. It acts along the line joining them.
- **Static charges:** Charged objects which have no relative movement.
- **Moving charges:** When charges or charged objects change their position with respect to their surroundings.
- **Electric current:** The rate of flow of charge in a conductor.
- **Electric field lines:** It is a curve, the tangent to which a point gives the direction of the magnetic field at that point.
- **Compass needle:** A freely suspended magnet, allowed rotating about a vertical axis. It rests in the north south direction .It deflects in the magnetic field due to magnets near it.
- **Magnetic field:** A region of influence around a magnet where its effect can be observed.

Introduction

We have studied in the previous lessons that a charge at rest produces an electric field. The relation between electricity and magnetism was first noticed by an Italian jurist *Gian Demenico Romagnosi* in 1802.

He found that an electric current flowing in a wire affects a magnetic needle, and published his observations in a local newspaper, *Gazettadi Jrentime*. However, his observations were overlooked. The fact that magnetic field is intimately related with an electric current was rediscovered in 1820 by a Danish Physicist, *Hans Christian Oersted*. His path breaking observation established a connection between moving charges and magnetic field.

Concept of Magnetic field

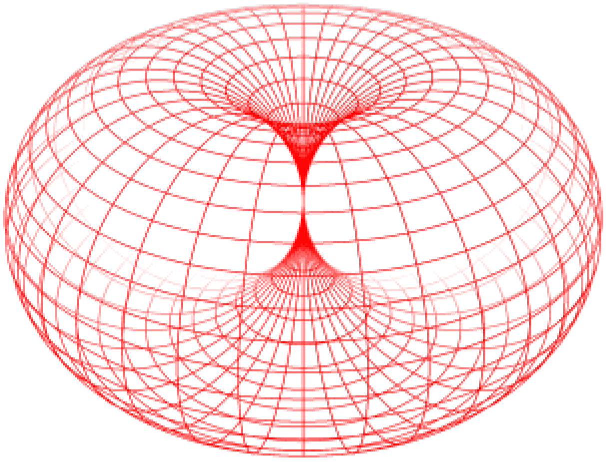


If we place or suspend a small magnetic needle near a bar-magnet, the needle rests in a definite direction. (Figure1). If we place this needle at some other point, it rests in some other direction. This shows that the magnetic compass needle, near a bar magnet experiences a torque which turns the needle to a definite direction.

(Figure 1)

The region near a magnet, where a magnetic needle experience a torque and rests in a definite direction, is called the "magnetic field" of the magnet

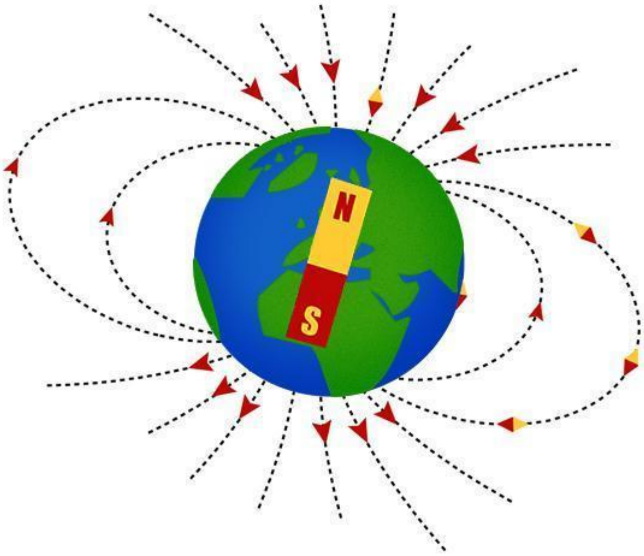
Magnetic field are in three dimensional space



https://gifer.com/en/O7lQ

The line drawn from the south to the north pole of a magnetic needle freely-suspended at a point in the magnetic field is the direction of the field at that point.

Earth behaves as though it has a magnet inside it, (this is not true) however, because of this magnet it has a magnetic field. This is why a freely-suspended magnetic needle always rests in the north-south direction. The north pole of the needle points towards north and the South Pole towards south. This shows that the earth's magnetic field acts from south to north.



https://www.google.com/search?site=imghp&tbm=isch&q=magnetic%20field%20&tbs=sur:fmc#imgrc=5QPQolBNxUyhoM:

Oersted Experiment

Oersted in 1820, found experimentally that a magnetic field is established around a current carrying conductor just as it occurs around a magnet. His experiment is shown in Figure-2.

A conducting wire AB is connected to the poles of a battery through a key. The wire is kept along a magnetic needle parallel to it (in north-south direction). It was observed

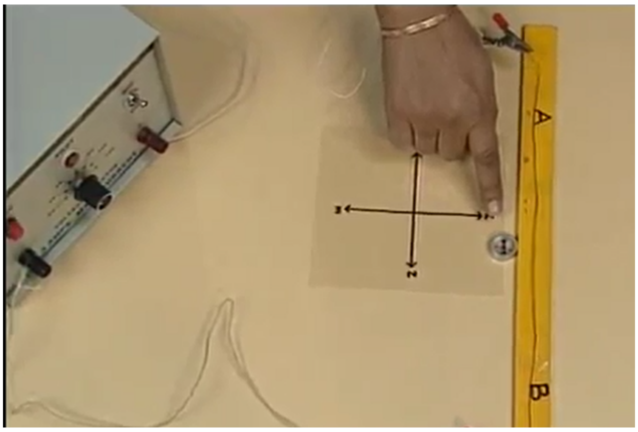
- So long as there is no current in the wire; the magnetic needle remains parallel to the wire (fig.2a).
- As soon as the key is pressed, a current flows through the wire and the needle is deflected (fig. 2b).
- If the current is reversed, the needle indicates that with the passage of current in the wire, a magnetic field is established around it.
- On increasing the current in the wire or on bringing the needle closer to the wire, the deflection of the needle increases.

This experiment shows that the magnetic field is produced due to the electric current since electric current is ‘charge in motion’, it is concluded that moving charges produce magnetic fields in the surrounding space. (A charge, whether stationary or in motion, produces an electric field around it. If it is in motion, then, in addition to the electric field, it also produces a magnetic field.

http://epathshala.nic.in/e-pathshala-4/e-resources-2/?opt_type=video&opt_lang=English&opt_class=Secondary&searching_text=magnetic+effect+of+current&search=Search

Watch the videos to understand the magnetic effect of current.

You will also recall the concepts you learnt while studying magnetic effect of current in your earlier class



Magnetic effect of current

Oersted Experiment

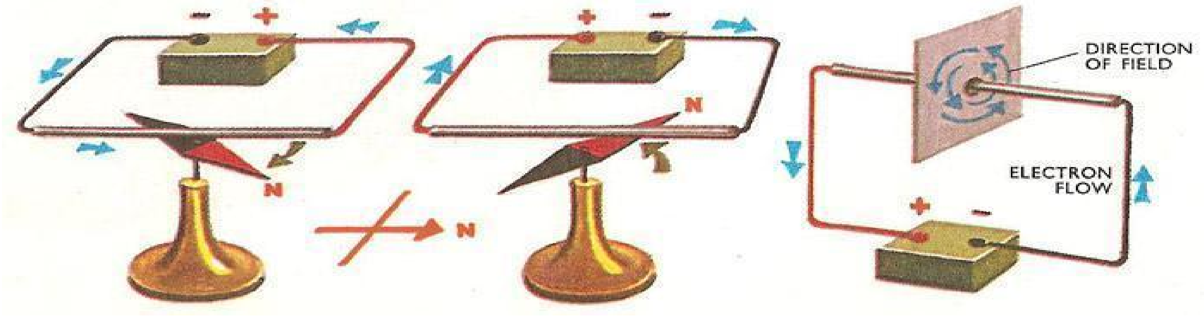
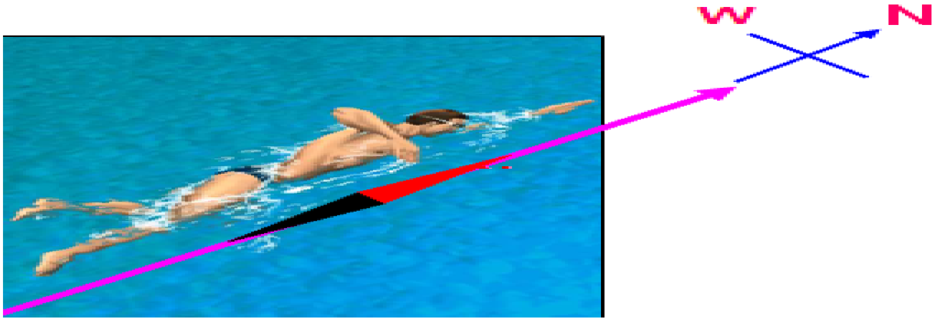


Figure: 2(a)

Figure: 2(b)

Figure: 2(c)

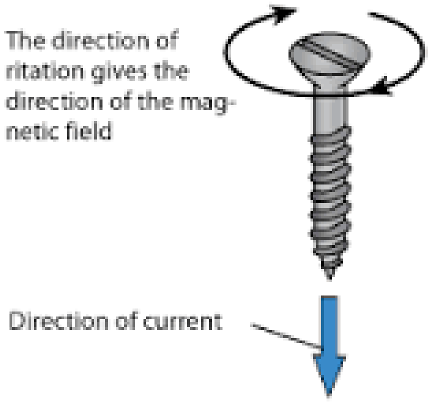
Ampere's Swimming Rule



This rule predicts the direction of deflection of the magnetic needle in the Oersted's experiment; it can be stated as follows:

"Imagine a man swimming along the wire in the direction of the flow of the current with his face always turned towards the magnetic needle, then the north pole of the needle will get deflected towards his left hand"as shown in the given image.

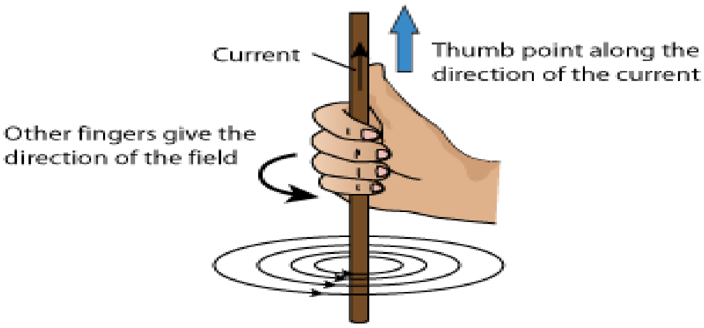
Maxwell's Cork Screw Rule or Right Hand Screw Rule



If the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor as shown in figure.

Right Hand Thumb Rule or Curl Rule

If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the fingers encircling the conductor will give the direction of the magnetic lines of force.



Right Hand Grip Rule

Biot-Savart Law

Oersted’s experiment showed that a current carrying conductor generates a magnetic field around it. French scientists *Biot* and *Savart*, in the same year 1820, performed a series of experiments to study the magnetic fields generated by various current carrying conductors and formulated a law to determine the magnitude and direction of the fields generated. This law is known as “Biot-Savart Law”.

Let us think about these

On what factors does the magnetic field depend upon?

a) Current in the conductor
b) Direction of current in the conductor
c) Distance of the observation point where we wish to find the value of magnetic field
d) Orientation of the current carrying conductor
e) Medium between the current carrying conductor and the observation point

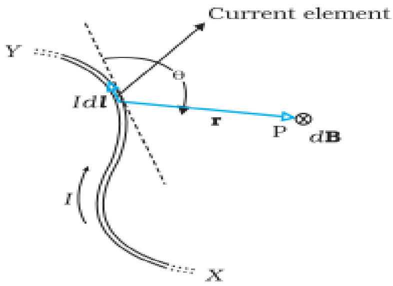
Let us visualize

All magnetic fields that we know are due to currents (or moving charges) and due to intrinsic magnetic moments of particles. Here, we shall study the relation between current and the magnetic field it produces.

It is given by Biot-Savart's law.

Given figure shows a finite conductor XY carrying current I. Consider an infinitesimal element dl of the conductor.

The magnetic field dB due to this element is to be determined at a point P which is at a distance r from it. Let θ be the angle between dl and the displacement vector r.



According to Biot-Savart’s law, **the magnitude of the magnetic field dB is proportional to the current I, the element length |dl|, and inversely proportional to the square of the distance r.**

Its direction is perpendicular to the plane containing **dl and r.**

Thus, in vector notation:

$$dB \propto \frac{I\, dl \times r}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \times r}{r^3}$$

If we use the property of cross-product.

$$dB = \frac{\mu_0 I\, dl\, sin\theta}{4\pi r^2}$$

Where $\mu_o/4\pi$ is a (dimensional) proportionality constant when we consider the medium around the wire to be vacuum.

The proportionality constant in SI units has the exact value:

$$\frac{\mu_0}{4\pi} = 10^{-7}\, Tm/A$$

We call μ_0 is the **permeability of free space (or vacuum).**

Its dimensions are **[$MLT^{-2}A^{-2}$].**

The resultant field at *P* due to the whole conductor can be found by integrating for the entire length of the conductor.

That is: $B=\int dB$

Permittivity and Permeability

In the universal law of gravitation, we say that any two point masses exert a force on each other which is proportional to the product of the masses m_1 , m_2 and inversely proportional to the square of the distance r between them.

We write it as

$$F = G\frac{m_1 m_2}{r^2}$$

where G is the universal constant of gravitation.

Similarly, in Coulomb's law of electrostatics we write the force between two point charges q_1, q_2 separated by a distance r as:

$$F = k\frac{q_1 q_2}{r^2}$$

where k is a constant of proportionality.

In SI units, k is taken as $1/4\pi\varepsilon$ where ε is the Permittivity of the medium.

Also in magnetism, we get another constant, which in SI units, is taken as $\mu/4\pi$ where μ is the Permeability of the medium.

Although G, ε and μ arise as proportionality constants, there is a difference between gravitational force and electromagnetic force.

While the gravitational force does not depend on the intervening medium, the electromagnetic force depends on the medium between the two charges or magnets.

Hence

- While G is a universal constant, ε and μ depend on the medium.
- They have different values for different media.
- The product εμ turns out to be related to the speed v of electromagnetic radiation in the medium through $\varepsilon\mu = 1/v^2$.

Electric permittivity ε is a physical quantity that describes how an electric field affects and is affected by a medium. It is determined by the ability of a material to polarize in response to an applied field, and thereby to cancel, partially, the field inside the material.

Similarly, magnetic permeability μ is the ability of a substance to acquire magnetization in magnetic fields. It is a measure of the extent to which magnetic fields can penetrate matter.

In vacuum relation between Permeability (μ_0) and Permittivity of free space (ϵ_0)

We know that:

$$\frac{\mu_0}{4\pi} = 10^{-7} NA^{-2}$$

and

$$\frac{1}{4\pi\epsilon_0} = 9\times10^{9} Nm^{2}C^{-2}$$

Dividing we get:

$$\mu_0\epsilon_0 = \frac{1}{\left(3\times10^{8}\right)^2}\left(\frac{C}{Am}\right)^2$$

But 1 C = 1 As

Therefore $\mu_0\epsilon_0 = \frac{1}{\left(3\times10^{8}ms^{-1}\right)^2}$

But $3\text{x}10^8$ m/s is the speed of light in free space (c).

Therefore $\mu_o\,\epsilon_0 = 1/c^2$

Or

$$c = 1/\sqrt{\mu_o\,\epsilon_0}$$

This is an interesting relation between ε_0, the permittivity of free space; μ_0, the permeability of free space; and c, the speed of light in vacuum:

Since the speed of light in vacuum is constant, the product $\mu_0\,\varepsilon_0$ is fixed in magnitude. Choosing the value of either ε_0 or μ_0, fixes the value of the other.

You can watch this video

https://www.youtube.com/watch?v=Joy5NYkXV14

(the video goes over Biot Savart's Law)

Comparison of Biot -Savart law and Coulomb's law

A stationary charge generates an electric field while a current generates a magnetic field in the surrounding space. The Coulomb's law gives the electric field due to a distribution of charges, while the Biot-Savart law gives the magnetic field due to a current element.

According to Coulomb's the magnitude of electric field at a point P (say) due to a charge element *dq* is

$$dE = dq/4\pi r^2$$

where *r* is the distance of *P* from the charge element.

According to Biot-Savart Law, the magnitude of magnetic field at P due to a current element *I dl* distant *r* from *P* is

$$dB = \mu_0 I\, dl \sin\theta / 4\pi r^2$$

Where 'ɵ' is the angle between the length of the element and the line joining the element to the point *P*.

We see that the Biot-Savart Law is the magnetic equivalent of Coulomb's Law and both are inverse square laws.

The two laws, however, differ in certain respects:

i) The charge element *dq* is a scalar, whereas current element *idl* is a vector whose direction is in the direction of current. According to Coulomb's Law, the magnitude of electric field depends only upon the distance of the charge element from the point *P*. According to Biot-Savart Law, the magnitude of magnetic field at *P* also depends upon the angle between the current element and the line joining the current element to the point *P*.

ii) According to Coulomb's law, the direction of electric field is along the line joining the charge element and the point *P*. According to Biot-Savart Law, the direction of magnetic field is perpendicular to the current element as well as to the line joining the current element to the point *P*.

Special Cases:

Case 1. If $\theta = 0^0$, $\sin\theta = 0$, so $dB = 0$

That is, the magnetic field is zero at points on the axis of the current element.

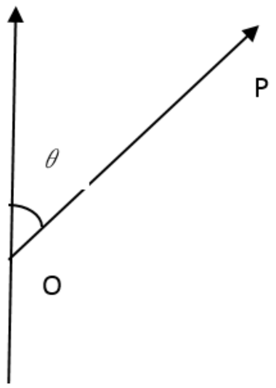
Case 2. If $\theta = 90^0$, $\sin\theta = 1$, so that dB = maximum i.e.

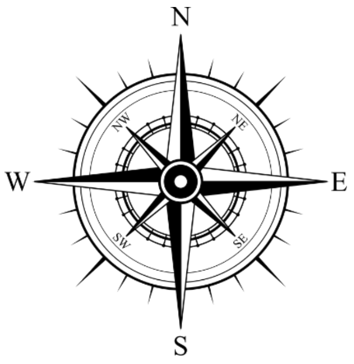
The magnetic field due to a current element is maximum in a plane passing through the element and perpendicular to its axis.

Examples based on Biot-Savart law

Example

A wire placed along the north-south direction carries a current of 8A from south to north. Find the magnetic field due to a 1 cm piece of wire at a point 200 cm north-east from the piece.





Solution

The problem is illustrated in figure above

As the distance *OP* is much larger than the length of the wire, we can treat the wire as a small current element.

Here $I = 8$ A, $dl = 1$ cm $= 10^{-2}$ m, $r = 200$ cm $= 2$m $\theta = 45^0$

$$dB = \mu_0 I\, dl \sin\theta / 4\pi r^2$$

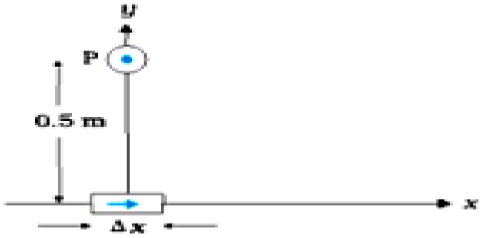
$dB = 1.4 \times 10^{-9}$ T

The direction of the magnetic field at point *P* is normally into the plane of the paper.

Example

An element $dl = x$ is placed at the origin and carries a large current $I = 10$ A

What is the magnetic field on the y-axis at a distance of 0.5 m. $x = 1$ cm.



Solution

$dB = \mu_0 \, Idl \sin \theta / 4\pi r^2$

$dl = x = 10^{-2}$ m, $I = 10$ A, $r = 0.5$ m $= y$, $\mu_0 / 4\pi = 10^{-7}$ Tm / A

$\theta = 90^0$, $\sin \theta = 1$

On substitution the values

$dB = 4 \times 10^{-8}$ T

The direction of the field is in the + Z direction. This is so since,

$dl \times r = \hat{i} \times \hat{j} = \hat{k}$

Summary

In this module we have learnt

- There is a magnetic field around a current carrying wire.
- The direction of magnetic field can be given by amperes swimming rule, right hand grip rule.
- The strength and direction can be given by Biot Savart's law.
- Vector form of Biot-Savart's law is:

  $$dB \propto \frac{Idl \times r}{r^3}$$

  $$dB = \frac{\mu_0}{4\pi} \frac{Idl \times r}{r^3}$$

- Permeability and permittivity of free space are related by: $\mathbf{c = 1/\sqrt{\mu_o \epsilon_0}}$